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Natural convection along a vertical complex wavy surface

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Abstract

A previously proposed transformation has been applied to the natural-convection boundary layer along a complex vertical surface created from two sinusoidal functions, a fundamental wave and its first harmonic. The numerical results demonstrate that the additional harmonic substantially alters the flow field and temperature distribution near the surface. The conclusion that the averaged heat-transfer rate per unit-wetted wavy surface is less than that of a corresponding flat plate [L.S. Yao, Natural convection along a vertical wavy surface, ASME J. Heat Transfer 105 (1983) 465–468] has been confirmed for this more complex surface. On the other hand, the total heat-transfer rates for a complex surface are greater than that of a flat plate. The numerical results show that the enhanced total heat-transfer rate seems to depend on the ratio of amplitude and wavelength of a surface.

1. Introduction

Since the work of Schmidt and Beckman [1], natural convection has been one of the most important research topics in heat transfer [2,3]. The problem of natural or mixed convection along a sinusoidal wavy surface extended previous work to complex geometries [4–6], and has received considerable attention due to its relevance to real geometries. An example of such geometry is a "roughened" surface that occurs often in problems involving the enhancement of heat transfer.

The method of transformed coordinates was originally proposed in [4] as a tool to solve heat-transfer problems in the presence of irregular surfaces of all kinds. This method is not limited to heat-transfer problems, but is also applicable to other problems in engineering and science, including those with unsteady boundary conditions. Its generality has been demonstrated by its original application to the natural convection along a sinusoidal wavy surface [4] because such a surface can be viewed as an approximation to certain practical geometries of relevance in heat transfer. A vast amount of literature about convection along a sinusoidal wavy surface is available for different heating conditions and various kinds of fluids [7–32].

One of the reasons why a roughened surface is more efficient in heat transfer is its capability to promote fluid motion near the surface; in this way a complex wavy surface, a sum of two or more sinusoidal surfaces, is expected to promote a larger heat-transfer rate than a single sinusoidal surface. This complex geometry will promote a correspondingly complicated motion in the fluid near the surface; this motion is described by the nonlinear boundary-layer equations. This expectation is the basis of the current study even though only laminar natural convection is studied. The mathematical

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Nomenclature

a_1, a_2	wave amplitudes
g	gravitational acceleration
Gr	Grashof number, Eq. (3c)
ℓ	fundamental wavelength
Nu	Nusselt number
p	pressure
Pr	Prandtl number
s	surface length, Eq. (9)
T, θ	temperature
u, v	velocity components
x, y, r	coordinates
v	kinematic viscosity
β	thermal expansion coefficient
ρ	density
σ	surface geometry function, Eq. (1)
Superscripts	
$\bar{}$	dimensional quantity
$\widehat{}$	transform variables, Eq. (3)
Subscripts	
w	surface
∞	free stream
x	derivative with respect to x

formulation following [4] is presented in the next section. The numerical results for unit Prandtl number are explained in the third section. The major conclusion is that the *total* heat-transfer rate for a wavy surface of any kind is, in general, greater than that of the corresponding flat plate, and may be a function of the ratios of amplitudes to wavelengths of the surface. On the other hand, the result that the average heat-transfer rate per unit of wetted surface for a wavy surface is less than that of a flat plate, as reported in [4], is confirmed. The total heat-transfer rate is the more important factor in designing a heat-transfer surface.

2. Formulation of the problem

Following [4], the geometric model considered is a semi-infinite vertical complex plate whose surface is described by

$$\bar{y} = \bar{\sigma}(\bar{x}) = a_1 \cdot \sin\left(\frac{2\pi\bar{x}}{\ell}\right) + a_2 \cdot \sin\left(\frac{4\pi\bar{x}}{\ell}\right), \tag{1}$$

where ℓ is the fundamental wavelength. The mathematical formulation proposed in [4] applies to a surface of arbitrary shape; a sinusoidal surface was used as the specific example in the computations. Here the study is focused on a complex surface formed by a fundamental wave and its first harmonic, which is more complex than the single sinusoidal surface extensively studied in the past two decades. This particular complex surface can reveal more information about practical geometric effects. The surface temperature is held at T_w, warmer than the ambient temperature T_∞.

The exact dimensional governing equations in two-dimensional Cartesian coordinates $(\bar{x}, \bar{y})$ (see Fig. 1) are the Navier–Stokes equations and the energy equation. The dimensionless form of the equations, after ignoring terms of small orders in Gr, is

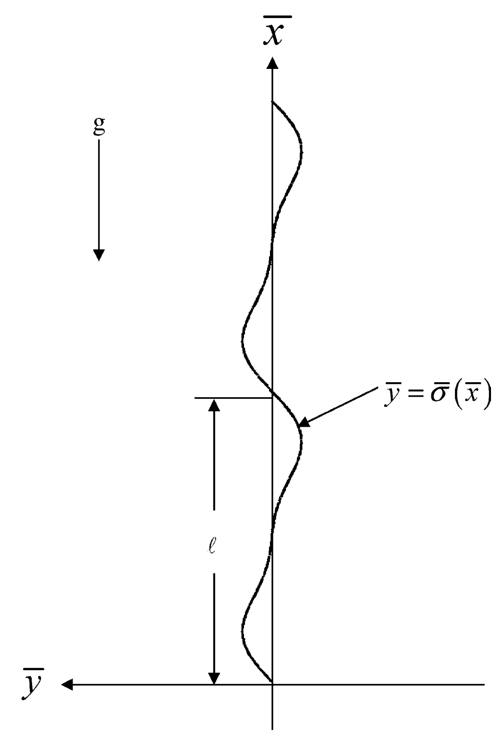

Fig. 1. Physical model and coordinates.

$$\frac{\partial \widehat{u}}{\partial x} + \frac{\partial \widehat{v}}{\partial r} = 0, \tag{2a}$$

$$\widehat{u}\frac{\partial \widehat{u}}{\partial x} + \widehat{v}\frac{\partial \widehat{u}}{\partial r} = -\frac{\partial p}{\partial x} + Gr^{1/4}\cdot\sigma_x\cdot\frac{\partial p}{\partial r} + \theta + (1+\sigma_x^2)\frac{\partial^2 \widehat{u}}{\partial r^2}, \tag{2b}$$

$$\sigma_{xx}\widehat{u}^2 + \sigma_x\theta = \sigma_x\frac{\partial p}{\partial x} - Gr^{1/4}\cdot(1+\sigma_x^2)\cdot\frac{\partial p}{\partial r}, \tag{2c}$$

$$\widehat{u}\frac{\partial \theta}{\partial x} + \widehat{v}\frac{\partial \theta}{\partial r} = \frac{(1+\sigma_x^2)}{Pr}\frac{\partial^2 \theta}{\partial r^2}, \tag{2d}$$

where

$$x=\frac{\bar{x}}{\ell},\quad r=\frac{\bar{y}-\bar{\sigma}}{\ell}\times Gr^{1/4}, \tag{3a}$$

$$\widehat{u}=\frac{\bar{u}\ell}{\nu Gr^{1/2}},\quad \widehat{v}=\frac{(\bar{v}-\bar{\sigma}_{\bar{x}}\bar{u})\ell}{\nu Gr^{1/4}},\quad p=\frac{\bar{p}\ell^2}{\rho\nu^2 Gr},\quad \theta=\frac{T-T_\infty}{T_w-T} \tag{3b}$$

and

$$Gr=\frac{\rho g\ell^3(T_w-T_\infty)}{\nu^2}, \tag{3c}$$

is the Grashof number. The transformation in (3a) maps the complex surface into a flat surface. The transformed coordinates (x,r) are not orthogonal, but a regular rectangular computational grid can be easily fitted onto the transformed coordinates. Problems associated with a complex geometry can then be studied by straightforward numerical methods. This is a practical method particularly for the study of heat-transfer enhancement, which usually deals with complex geometries. As is apparent from (3), the velocity components $(\widehat{u},\widehat{v})$ are neither parallel to nor perpendicular to the complex surface. The convection induced by the irregular surface is explicitly described in Eq. (2). Eq. (2c) indicates that the pressure gradient along the r-direction is $O(Gr^{-1/4})$. This implies that the lowest-order pressure gradient along the x-direction can be determined from the inviscid-flow solution as in traditional boundary-layer formulations. For a natural convection problem, this pressure gradient is zero. Eq. (2c) also shows that $Gr^{1/4}\frac{\partial p}{\partial r}$ is O(1) and is determined by the left-hand side of the equation. Elimination of $\frac{\partial p}{\partial r}$ between (2b) and (2c) results in three equations for u, v and θ. They are

$$(4x)\frac{\partial u}{\partial x}+2u-y\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}=0, \tag{4a}$$

$$(4x)u\frac{\partial u}{\partial x}+(v-yu)\frac{\partial u}{\partial y}+\left(2+\frac{4x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)u^2=\frac{\theta}{1+\sigma_x^2}+(1+\sigma_x^2)\frac{\partial^2 u}{\partial y^2}, \tag{4b}$$

$$(4x)u\frac{\partial\theta}{\partial x}+(v-yu)\frac{\partial\theta}{\partial y}=\frac{1+\sigma_x^2}{Pr}\frac{\partial^2 u}{\partial y^2}, \tag{4c}$$

where

$$x=\frac{\bar{x}}{l},\quad y=\frac{r}{(4x)^{1/4}}=\frac{\bar{y}-\bar{\sigma}}{\left(\frac{4\nu^2\bar{x}}{\beta g\Delta T}\right)^{1/4}},$$

$$u=\frac{\widehat{u}}{(4x)^{1/4}}=\frac{\bar{u}}{(4\beta g\Delta T\bar{x})^{1/2}},\quad v=(4x)^{1/4}\widehat{v}=\frac{(\bar{v}-\bar{\sigma}_{\bar{x}}\bar{u})}{\left(\frac{\beta g\Delta T\nu^2}{4\bar{x}}\right)}, \tag{5}$$

It should be noted that the normal length and velocity scales are independent of the wavelength. The axial length scale is the wavelength. This is the reason why there is no similarity solution for a complex surface. For a slightly curved surface, say, $\sigma\sim Gr^{-1/4}$, the curvature effects are negligible. This is a case for which the Prandtl transposition theorem can be readily applied [33]. For a wavy surface of finite amplitude or curvature, curvature effects are important and cannot be ignored.

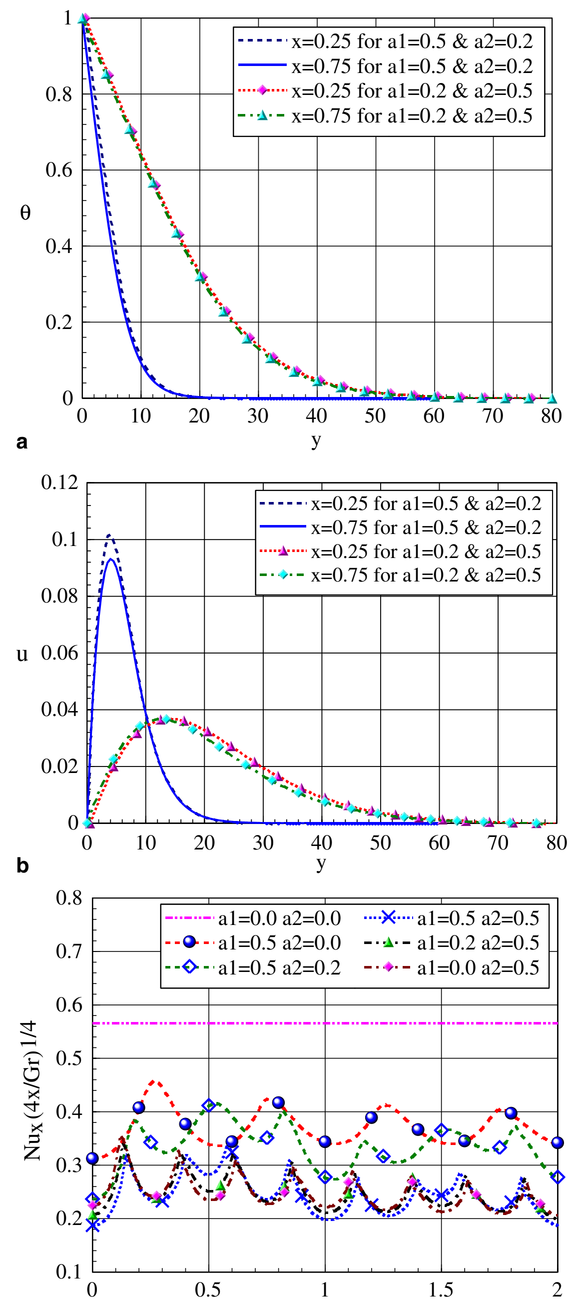

Fig. 2. (a) Temperature distribution, (b) axial velocity, (c) local heat transfer rate.

The associated boundary conditions are

(1) On the wavy surface ($y = 0$)

$$\theta = 1 \text{ (constant temperature), and}$$
$$u = v = 0 \text{ (no slip and no penetration)} \tag{6a}$$

(2) Matching with the quiescent free stream ($y \to \infty$)

$$\theta \text{ and } u \to 0 \tag{6b}$$

A finite-difference solution of (4) is straightforward since the computational grids are fitted to shape of the wavy surface. The central difference is used for the diffusion terms and the forward-difference scheme is used for the convection terms. The singularity at $x = 0$ has been removed by the scaling; therefore, the computation can be started at $x = 0$, and then marches downstream implicitly. After several tries, 0.005 and 0.01 are used for the x- and y-grids, respectively, and the size of the computational domain is $x = 2$ and $y = 80$. Such small grid sizes are probably over conservative, but can be easily handled by any personal computer today.

3. Results and discussion

Numerical results have been obtained for the Prandtl number one. In Fig. 2a, the temperature distribution is plotted for two cases, case 1 is $a_1 = 0.2$, $a_2 = 0.5$; and case 2 is $a_1 = 0.5$, $a_2 = 0.2$ at $x = 0.25$ and 0.75, respectively. For case 1, $x = 0.25$ and 0.75 are the nodes of the wavy surface, while $x = 0.25$ is the crest and $x = 0.75$ is the trough for the case 2. The axial velocity, whose direction is along the x-coordinate and nonparallel to the wavy surface, is plotted in Fig. 2b. They show that the size of the selected computational domain is sufficient large to satisfy the boundary condition (6b).

The local Nusselt number, defined in terms of $(T_w - T_\infty)$, thermal conductivity, k, and the fundamental wavelength, ℓ, can be expressed as

$$Nu_x(4x/Gr)^{1/4} = -(1+\sigma_x)^{1/2}\frac{\partial\theta}{\partial y}\bigg|_{y=0} \tag{7}$$

Eq. (7) is plotted in Fig. 2c for a_1 and $a_2 = 0$, 0.2, and 0.5, respectively. The shape of local heat-transfer rate is similar to those presented in [4]. An interesting new

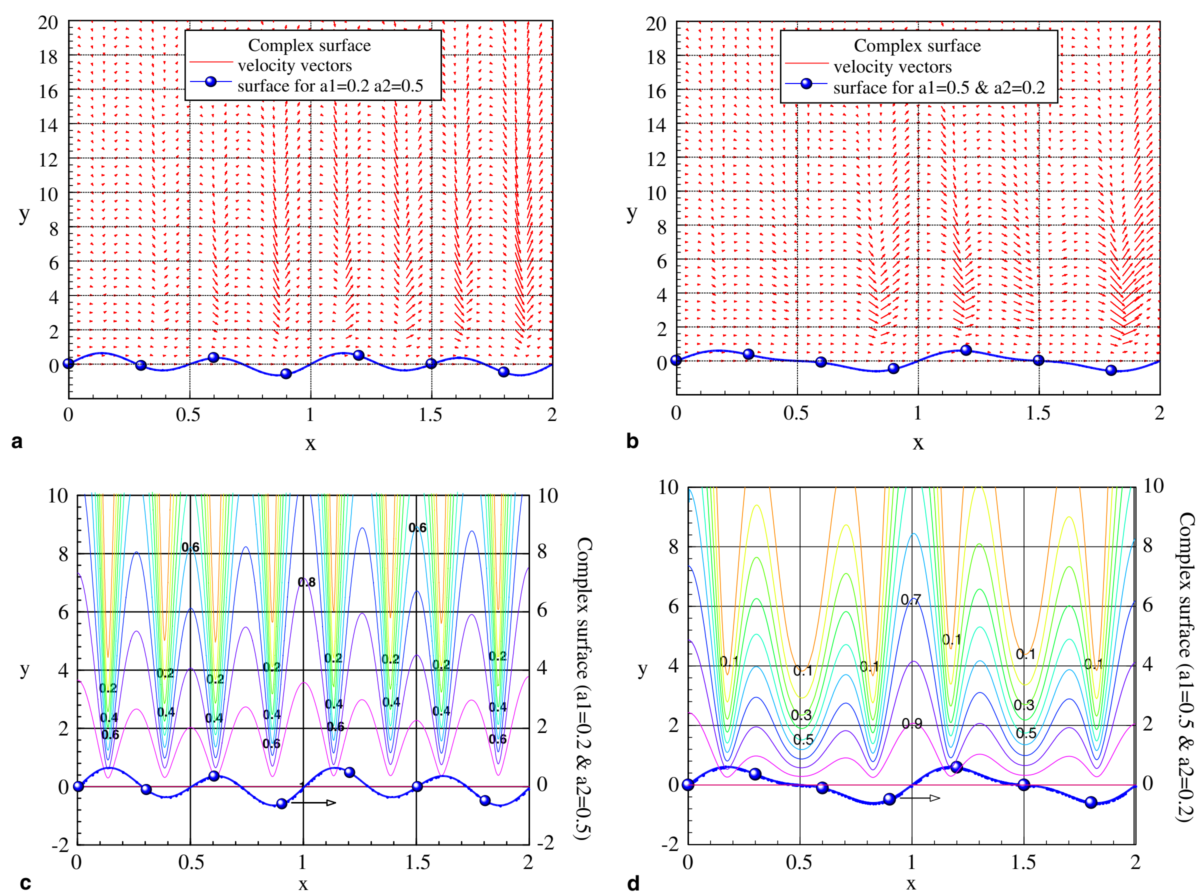

Fig. 3. Plot of velocity vectors of (a) case 1, (b) case 2. Plot of isothermal lines of (c) case 1, (d) case 2.

fact is that the local heat-transfer rate is insensitive to the amplitude, which is less than 0.5 of the fundamental wave when $a_2 = 0.5$. This is due to the fact that the flow velocity near the wavy surface determines the local heat-transfer rate. If the amplitude of the harmonic wave becomes dominant, the fundamental wave is a lesser factor in determining the surface curvature; consequently, the amplitude of the fundamental wave plays a lesser role in determine the local heat-transfer rate. This can be seen clearly with the plot of velocity vectors near the surface in Fig. 3. The wavy surface induces a local *approaching and leaving* flow slightly downstream from the trough as well as slightly upstream of the crest of the wavy surface. Such a flow pattern is the mechanism that enhances the local heat-transfer rate. For case 1, the wavelength of the wavy surface is $\ell/2$, so the heat transfer enhancement is twice as large as for the case 2. This can also be seen from the plot of isothermal lines near the surface in Fig. 3.

The averaged Nusselt number per unit-wetted surface area can be obtained by integrating Eq. (7)

$$Nu/Gr^{1/4}/s = \frac{1}{s}\int_0^x \left[\frac{1+\sigma_x^2}{(4x)^{1/4}} \cdot \frac{\partial\theta}{\partial y}\right]_{y=0} \cdot \mathrm{d}x, \tag{8}$$

where

$$s = \int_0^x (1+\sigma_x^2)^{1/2} \cdot \mathrm{d}x. \tag{9}$$

Eq. (8) is plotted in Fig. 4. It confirms what has been concluded in [4], that is, the averaged Nusselt number per unit surface area for a wavy surface is smaller than that of a corresponding flat plate. It is shown in Fig. 4a that the Nusselt number decreases when the amplitude of the harmonic wave increases while the amplitude of the fundamental wave is fixed at 0.5. A comparison of two cases, $a_1 = 0.5$, $a_2 = 0.0$ and $a_1 = 0.5$, $a_2 = 0.2$, shows that the ratio of wave amplitude and its wavelength may be an important parameter in determining the local heat-transfer rate. Thus, the ratio of the wave amplitude to wavelength changes when the amplitude of the harmonic wave increases, so does the averaged heat transfer rate shown in Fig. 4a. In Fig. 4b, the Nusselt number is insensitive to the increase of the amplitude of the fundamental wave while the amplitude of the harmonic wave is held at 0.5. This is because the variation of the amplitude of the fundamental wave whose magnitude is smaller than that of the harmonic wave has little effect on the ratio of amplitude to wavelength. This results that the ratio of amplitude to wavelength for the surface is determined by the harmonic wave when its amplitude is held at 0.5, and is not affected by changing the amplitude of the fundamental wave.

The total heat-transfer rate, Eq. (8) without division by s, is plotted in Fig. 4c. It shows that the total heat-

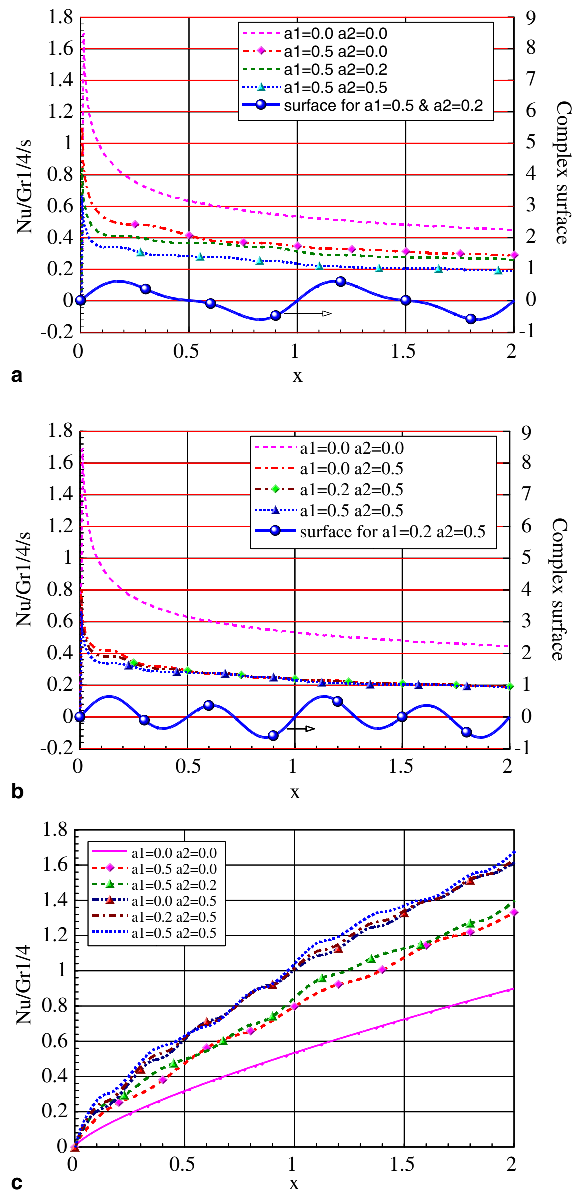

Fig. 4. (a) Total heat transfer rate per unit wetted area for various a_2 and $a_1 = 0.5$. (b) Total heat transfer rate per unit wetted area for various a_1 and $a_2 = 0.5$. (c) Total heat transfer rate.

transfer rate for the wavy surface is indeed much greater than that of a flat plate. The enhanced rate seems to be proportional to the ratio of amplitude to wavelength of the surface. The rate for $a_2 = 0.5$ and a_1 less than 0.5 is almost twice as much as the heat-transfer rate of a flat plate. This is a result that was not reported in [4]. It seems to convey an incorrect impression that a wavy surface is inferior in enhancing natural-convection heat transfer. The total heat-transfer rate is a more important factor in designing a heat-transfer surface than its averaged rate. It should be noted that the model study in this paper is limited to *laminar* natural convection. A wavy surface can trigger an early transition of a natural-convection boundary layer, which can further enhance the cooling capability of a wavy surface. Additional study is required to clarify this mechanism.

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